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THE CONTACT PROBLEM FOR AN ANISOTROPIC WEDGE-SHAPED PLATE WITH AN ELASTIC FASTENING OF VARIABLE STIFFNESS†

R. D. BANTSURI and N. N. SHAVLAKADZE

Tbilisi

e-mail: nusha@rmi.acnet.ge

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The two-dimensional contact problem (the case of plane stress) of the transfer of a shear force of given intensity to an elastic anisotropic wedge-shaped plate through an elastic rod of variable stiffness is considered. It is assumed that the rod is connected to one of the edges of the plate, the other edge is stress-free, and the bending stiffness of the rod is negligibly small. This problem is solved in closed form by reducing it to the Karleman boundary-value problem for a strip with a shift. Various cases of the variation of the stiffness of the supporting rod are studied. The nature of the contact shear stress at the vertex of the wedge is analysed. © 2002 Elsevier Science Ltd. All rights reserved.

Contact problems of the interaction between elastic bodies of various shapes (including wedge-shaped bodies) and thin elastic elements in the form of stringers or inclusions were considered in [1–3]. Problems for an elastic isotropic or anisotropic wedge, supported by a rod of constant stiffness [4–7], as well as the problem for an elastic isotropic wedge, supported along the bisector by an elastic rod of variable stiffness [8], have been studied by means of boundary-value problems of the theory of analytical functions.

An elastic anisotropic thin wedge-shaped plate occupying an angle $-\theta < \arg z < 0$, $0 < \theta < 2\pi$ in a plane is considered. One side of the angle $\arg z = -\theta$ is free and a rod of variable tensile stiffness is glued to the other side $\arg z = 0$. We will determine the distribution of the contact forces along the fastening line as well as the elastic equilibrium of the plate under a tangential load of intensity $\tau_0(x)$ applied along the rod. We will assume that the bending stiffness of the rod is negligibly small, i.e. $\sigma_y^0 = 0$.

From the equilibrium condition for any part $(0, x)$ of the rod we have

$$S_0(x)\sigma_x^0(x) - h\int_0^x [\tau_{xy}^0(s) - \tau_0(s)]ds = 0, \quad x > 0 \tag{1}$$

The condition for complete contact between the elastic rod and the wedge has the form (the prime denotes differentiation with respect to x)

$$u_0'(x) = u'(x, 0), \quad \tau_{xy}^0(x) = \tau_{xy}(x, 0) \equiv \tau(x), \quad x > 0 \tag{2}$$

By Hooke's law, taking into consideration that $\sigma_y^0 = \sigma_y = 0$, we have

$$u_0'(x) = \sigma_x^0(x)/E_0(x), \quad u'(x, 0) = a_{16}\tau_{xy}(x, 0) + a_{11}\sigma_x(x, 0) \tag{3}$$

Here $E_0(x)$ is the modulus of elasticity of the rod, a_{11} and a_{16} are the elasticity constants of the plate, $\sigma_x^0(x)$, $\tau_{xy}^0(x)$ and $\sigma_x(x, y)$ $\tau_{xy}(x, y)$ are the normal and shear stresses of the rod and the wedge, respectively, $u_0(x)$ and $u(x, y)$ are the horizontal displacements of the rod and elastic wedge, respectively; $s_0(x)$ is the cross-section area of the rod and h is the plate thickness.

Condition (1) can be written, taking Eqs (2) and (3) into account, in the following form

$$k_1(x)\sigma_x(x) + k_2(x)\tau(x) - hJ(x) = 0, \quad x > 0 \tag{4}$$

$$k_1(x) = s_0(x)E_0(x)a_{11}, \quad k_2(x) = s_0(x)E_0(x)a_{16}$$

$$J(x) = \int_0^x [\tau(s) - \tau_0(s)]ds$$

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The equilibrium condition for the rod has the form

$$J(\infty)=0 \tag{5}$$

Consider two planes of the complex variables: $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$, which are obtained from the plane $z = x + iy$ by the affine transformations $x_n = x + \alpha_n y$ and $y_n = \beta_n y$, $\beta_n > 0$, respectively, where $s_n = \alpha_n + i\beta_n$ ($n = 1, 2$) are the roots of the characteristic equation, where $s_1 \neq s_x$ [9].

The given domain $S(-\theta < \arg z < 0)$ in the plane of the complex variable z is mapped by means of these transformations into the domains $S_n(-\theta_n < \arg z_n < 0)$, respectively, in the plane z_n ($n = 1, 2$) where

$$\operatorname{tg}\theta_n = \beta_n \sin\theta(\cos\theta - \alpha_n \sin\theta)^{-1}, \quad 0<\theta_n<2\pi$$

The problem is thus reduced, by means of well-known relations [9] expressing the components of the stress vector in terms of two analytical functions, to solving the following boundary-value problem of the theory of functions of a complex-variable: it is required to find two functions $\Phi_1(z_1)$ and $\Phi_2(z_2)$ that are analytical in the domains S_1 and S_2, respectively, with the following boundary conditions

$$(s_1-\bar{s}_2)t_1\Phi_1(t_1)+(\bar{s}_1-\bar{s}_2)\bar{t}_1\overline{\Phi_1(t_1)}+(s_2-\bar{s}_2)t_2\Phi_2(t_2)=0 \tag{6}$$

$$t_n=\rho(\cos\theta-s_n\sin\theta), \quad \rho=|t|\geqslant 0$$

$$(s_1-\bar{s}_2)\Phi_1(t_1)+(\bar{s}_1-\bar{s}_2)\overline{\Phi_1(t_1)}+(s_2-\bar{s}_2)\Phi_2(t_2)=-\tau(x) \tag{7}$$

$$t_1=t_2=x>0$$

$$2\operatorname{Re}[k_1(x)a\Phi_1(x)]+[k_2(x)-2\alpha_2 k_1(x)]\tau(x)=hJ(x), \quad x>0 \tag{8}$$

$$a=(s_1-s_2)(s_1-\bar{s}_2)$$

We will assume that the stresses and rotations vanish at infinity and hence consider that for large $|z_n|$

$$\Phi_n(z_n)=\gamma_n/z_n+O(1/z_n), \quad n=1,2$$

We will also assume that the functions $\Phi_1(z_1)$ and $\Phi_2(z_2)$ are continuously extended to all boundary points, except possibly the points $z_n = 0$, at which they satisfy the following conditions

$$\lim z_n\Phi_n(z_n)=0 \text{ when } z_n\to 0$$

Thus we will search for the functions $\Phi_1(z_l)$ and $\Phi_2(z_2)$ in the form

$$\Phi_n(z_n)=\frac{1}{\sqrt{2\pi}z_n}\int_{-\infty}^{\infty}\frac{A_n(t)}{t}e^{it\ln z_n}dt-\frac{a_n}{z_n}, \quad z_n\in S_n \tag{9}$$

where

$$a_n=\lim_{z_n\to 0}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{A_n(t)}{t}e^{it\ln z_n}dt, \quad n=1,2 \tag{10}$$

At the point $t = 0$ the integrals are considered in the sense of the principal Cauchy value. It can be shown that $a_n = -i\sqrt{\pi/2}A_n(0)$ from which it follows that $\gamma_n = -2a_n = i\sqrt{2\pi}A_n(0)$. We can also conclude from Eqs (6) and (9) that a_1 and a_2 satisfy the condition

$$(s_2-\bar{s}_2)a_2=(\bar{s}_2-s_1)a_1+(\bar{s}_2-\bar{s}_1)\bar{a}_1$$

Substituting Eq. (9) into conditions (6) and (7), carrying out a Fourier transformation and solving the last system for $A_n(t)$ ($n = 1, 2$), we obtain

$$A_1(t)=\frac{1}{2\Delta(t)}\left[(\bar{s}_1-s_2)e^{-\delta t}+(\bar{s}_2-\bar{s}_1)e^{-\gamma t}+(s_2-\bar{s}_2)e^{-i\mu t}\right]tT(t) \tag{11}$$

$$\Delta(t)=|s_1-s_2|^2\operatorname{ch}\gamma t-|s_1-\bar{s}_2|^2\operatorname{ch}\delta t+4\beta_1\beta_2\cos\mu t$$

$$T(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{s}\tau(e^{s})e^{-its}dt$$

$$\gamma=\theta_1+\theta_2,\quad \delta=\theta_1-\theta_2,\quad \mu=\ln|\cos\theta-s_1\sin\theta|-\ln|\cos\theta-s_2\sin\theta|$$

The function $A_2(t)$ is obtained from the expression for $A_1(t)$ by interchanging s and s_2 and θ_1 and θ_2.

It is obvious that $\overline{T(-t)}=T(t)$. Since the stress vanishes at infinity, taking the limit in the relation for $T(t)$ we obtain

$$T(0)=T_0/\sqrt{2\pi},\quad T_0=\int_0^{\infty}\tau(t)dt=\int_0^{\infty}\tau_0(t)dt$$

It can be proved that the function $\Delta(t)$ does not vanish anywhere for real t, apart from the point $t=0$, where it has a double zero root. The function in square brackets in the equation for $A_1(t)$ behaves similarly. Consequently, if the function $\tau(x)$ is absolutely integrable, the functions $A_1(t)$ and $A_2(t)$ will be continuous along the whole axis. Thus from Eq. (11) it follows that

$$A_1(0)=\frac{(\bar{s}_1-\bar{s}_2)\gamma-(\bar{s}_1-\bar{s}_2)\delta-i\mu(s_2-\bar{s}_2)}{|s_1-s_2|^2\gamma^2-|s_1-\bar{s}_2|^2\delta^2-4\beta_1\beta_2\mu^2}\frac{T_0}{\sqrt{2\pi}} \tag{12}$$

Hence the constants a_1, a_2, γ_1 and γ_2 are determined.

By substituting the value of the function $\Phi_1(z_1)$, determined by Eqs (9) and (11), into the boundary condition (8), by the Vieta formulae for characteristic equations, we get

$$\frac{1}{\sqrt{2\pi}i}\int_{-\infty}^{\infty}\frac{\Delta_1(t)}{\Delta(t)}T(t)e^{it\ln x}dt-\frac{hx}{k_1(x)}J(x)=2\operatorname{Re}aa_1 \tag{13}$$

$$\Delta_1(t)=-(\beta_1+\beta_2)|s_1-s_2|^2\operatorname{sh}\gamma t+(\beta_1-\beta_2)|s_1-\bar{s}_2|^2\operatorname{sh}\delta t+4|\alpha_1-\alpha_2|\beta_1\beta_2\sin\mu t$$

Let $k_1(x)=d_0x^{\alpha}$, $d_0>0$ and let α be any real number. After substituting $\ln x=\xi$, Eq. (13) becomes

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{G(t)}{t}T(t)e^{it\xi}dt-He^{-k\xi}\left(\int_{-\infty}^{\xi}[\tau(e^{s})-\tau_0(e^{s})]e^{s}ds\right)=2\operatorname{Re}aa_1 \tag{14}$$

$$G(t)=\frac{\Delta_1(t)}{\Delta(t)}t,\quad k=\alpha-1,\quad H=\frac{h}{d_0}$$

Differentiating both sides of Eq. (14) and applying an inverse Fourier transformation to the relation obtained, with the complex variable $t=t_0-i\varepsilon$ as a parameter (ε is as small a positive number as desired), we obtain

$$G(t)\Psi(t)-H\Psi(t-ik)=F(t),\quad -\infty-i\varepsilon<t<+\infty-i\varepsilon \tag{15}$$

$$t\Psi(t)=T(t)-T_0(t),\quad F(t)=-\frac{G(t)T_0(t)}{t}$$

$$T_0(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{s}\tau_0(e^{s})e^{-its}ds$$

Suppose $k>0$. The problem considered is reduced to the following problem of the Karleman type for a strip: obtain the function $\Psi(z)$, which is holomorphic in the strip $-k-\varepsilon<\operatorname{Im}z<-\varepsilon$, vanishes at infinity, is continuously extendable on the strip boundary and satisfies condition (15).

Using results obtained earlier [10], the function $\Psi(z)$ can be represented in the following form

$$\Psi(z)=\frac{\chi(z)}{2ikH}\int_{-\infty-i\varepsilon}^{+\infty-i\varepsilon}\frac{F(t)}{\chi(t-ik)}\left(\operatorname{sh}\frac{\pi}{k}(t-z)\right)^{-1}dt,\quad -k-\varepsilon<\operatorname{Im}z<-\varepsilon \tag{16}$$

$$\chi(z)=\frac{1}{z}\chi_k(z)\varkappa(z)\operatorname{sh}\frac{\pi}{2k}z,\quad \varkappa(z)=k^{iz/k}\Gamma\left(\frac{k+iz}{k}\right)\exp(iz\ln H_0^{1/k})$$

$$\chi_k(z) = \exp\left\{\frac{1}{2ik}\int_{-\infty-i\varepsilon}^{+\infty-i\varepsilon} \ln G_k(t)\,\mathrm{cth}\frac{\pi}{k}(t-z)dt\right\}$$

$$G_k(t) = -\frac{\Delta_1(t)}{(\beta_1+\beta_2)\Delta(t)}\mathrm{th}\frac{\pi}{2k}t, \quad H_0 = \frac{\beta_1+\beta_2}{H}$$

Suppose $k \geqslant 1$. If the function $T_0(z)$ is analytically extendable in the strip $-1 < \mathrm{Im}\, z < 1$ and vanishes exponentially at infinity, it follows from condition (15) and Eq. (16) that the function

$$\Psi_1(z) = \begin{cases} \Psi(z), & -k-\varepsilon < \mathrm{Im}\, z < -\varepsilon \\ [F(z) + H\Psi(z-ik)]/G(z), & -\varepsilon < \mathrm{Im}\, z < k-\varepsilon \end{cases}$$

is holomorphic in the strip $-k - \varepsilon < \mathrm{Im}\, z < k - \varepsilon$, vanishes exponentially at infinity, is bounded in the entire strip with the exception of the points $z_j^+ = t_j^+ + i\tau_j^+$ ($j = 0, 1\ldots p$), which are the zeroes of the function $G(z)$ in the upper strip.

Thus, according to Cauchy's formula, the required contact stress can be represented in the following form

$$\tau(x) - \tau_0(x) = \frac{x^{-1}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} t\Psi(t)e^{it\ln x}dt = \frac{x^{-1}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(t-ik)\Psi(t-ik)e^{i(t-ik)\ln x}dt$$

Consequently, in the vicinity of the vertex of the angle (as $x \to 0$) we will obtain $\tau(x) - \tau_0(x) = x^{k-1}\varphi_0(x)$, where $\varphi_0(x)$ is a bounded function near the point $x = 0$. For large x we get

$$\tau(x) - \tau_0(x) = O(1/x^{1+\tau_0^+})$$

If $0 < k < 1$, the function $\Psi_{(z)}$, given by Eq. (16), is analytically continuous in the strip $-1 < \mathrm{Im}\, z < 1$, apart from the points $\omega_j^- = \lambda_j^- + i\mu_j^-$ ($j = 0, 1, l$), which are the poles of the function $G_{(z)}$ in this strip. Then shear stress near the point $x = 0$ is represented as follows:

$$\tau(x) - \tau_0(x) = \frac{x^{-1}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(t-i)\Psi(t-i)e^{i(t-i)\ln x}dt +$$

$$+\frac{x^{-1}}{\sqrt{2\pi}}\mathrm{res}[z\Psi(z)e^{iz\ln x}]_{\omega_0^- = \lambda_0^- + i\mu_0^-} = c_1 x^{-(\mu_0^- + 1)} + \varphi_1(x), \quad c_1 = \mathrm{const}$$

where $\varphi_1(x)$ is a bounded function for $x \geqslant 0$.

We will now consider the case when $k < 0$ ($\alpha < 1$), i.e. the stiffness of the rod increases at the vertex of the angle and vanishes at infinity, and the entire principal vector of external load is transferred to the wedge. Putting $m = -k$, when can write condition (15) in the following form

$$G(t)\Psi_0(t) - H\Psi_0(t+im) = F(t), \quad -\infty - i\varepsilon < t < +\infty - i\varepsilon \tag{17}$$

Consider the following problem: it is required to find a function $\Psi_2(z)$ which is holomorphic in the strip $-m - \varepsilon < \mathrm{Im}\, z < m - \varepsilon$, vanishes at infinity and is bounded in the entire strip, apart from the points $z_j^- = t_j^- + i\tau_j^-$ ($j = 0, 1, \ldots, q$) which are the zeroes of the function $G_{(z)}$ in the lower half-space.

If the following problem is solved: find the function $\Psi_0(z)$, which is holomorphic in the strip $-\varepsilon < \mathrm{Im}\, z < m - \varepsilon$, vanishes at infinity and is continuously extendable at the strip boundary due to the boundary condition (17), then the solution of the previous problem will be the function

$$\Psi_2(z) = \begin{cases} \Psi_0(z), & -\varepsilon < \mathrm{Im}\, z < m-\varepsilon \\ [F(z) + H\Psi_0(z+im)]/G(z), & -m-\varepsilon < \mathrm{Im}\, z < -\varepsilon \end{cases}$$

Using the results obtained previously [10], the function Ψ_0 can be represented in the form

$$\Psi_0(z) = -\frac{\tilde{\chi}(z)}{2imH}\int_{-\infty-i\varepsilon}^{+\infty-i\varepsilon}\frac{F(t)}{\tilde{\chi}(t+im)}\left(\mathrm{sh}\frac{\pi}{m}(t-z)\right)^{-1}dt, \tag{18}$$

$$\tilde{\chi}(z) = \frac{1}{z}\chi_m(z)\tilde{\varkappa}(z)\,\mathrm{sh}\frac{\pi}{2m}z, \quad \tilde{\varkappa}(z) = m^{-iz/m}\Gamma\left(\frac{m-iz}{m}\right)\exp(-iz\ln H_0^{1/m})$$

If $\tau_0^- < 1$, then the function $\Psi_2(z)$ is analytically extendable in the strip $-1 < \text{Im}\, z < m - \varepsilon$ and the shear stress $\tau(x) - \tau_0(x)$ is bounded at the point $x = 0$. If $\tau_0^- > -1$, then the function $\Psi_2(z)$ has the pole closest to the real axis at the point $z_0^- = t_0^- + i\tau_0^-$, the function $T(t) - T_0(t)$ has a similar character and the unknown contact stress near the point $x = 0$ can be represented in the form

$$\tau(x) - \tau_0(x) = c_2 x^{-(\tau_0^- + 1)} + \varphi_2(x)$$

For large x we have

$$\tau(x) - \tau_0(x) = O(1/x^{1+m})$$

If $\alpha = 1$ ($k = m = 0$), condition (17) gives

$$\Psi(z) = F(z)/(G(z) - H)$$

and the shear stress has the form

$$\tau(x) - \tau_0(x) = O(x^{\lambda - 1}) \quad \text{as } x \to 0, \quad \lambda = \text{Im}\,\mu$$

where μ is chosen from the zeros closest to the real axis of the functions $\Delta(z)$ and $G(z) - H$ in the lower half-space.

For $\alpha < 1$, when $\theta = \pi$, i.e. the anisotropic body is a half-plane, the function

$$G(z) = -(\beta_1 + \beta_2) z \,\text{cth}\, \pi z$$

has a single pure imaginary root $z_0 = -i/2$ in the strip $-1 < \text{Im}\, z < 0$, and the shear stress near point $x = 0$ has the form

$$\tau(x) - \tau_0(x) = c_2 x^{-1/2} + \varphi_2(x)$$

When $\theta = 2\pi$, i.e. the body occupies the entire plane, cut along the positive part of the real axis, then

$$G(z) = -(\beta_1 + \beta_2) z \,\text{cth}\, 2\pi z$$

This function has pure imaginary roots $z_0 = -i/4$, $z_1 = -3i/4$ in the strip $-1 < \text{Im}\, z < 0$, and the shear stress as $x \to 0$ has the following form:

$$\tau(x) - \tau_0(x) = c_3 x^{-3/4} + c_4 x^{-1/4} + \varphi_3(x)$$

Here $\varphi_2(x)$ and $\varphi_3(x)$ are bounded functions for $x \geqslant 0$ and c_2, c_3 and c_4 are constants.

For $1 < \alpha \leqslant 2$, when $\theta = \pi$, the function $G_{(z)}$ has a pole at the point $\omega_0^- = -i$, the shear stress is bounded in the vicinity of the vertex of the angle. When $\theta = 2\pi$ as $x \to 0$ the shear stress has a singularity of the order of the square root.

Similar results are obtained in the case of an isotropic body [4].

Now consider an orthotropic body. Then

$$\Delta_1(t) = -(\beta_1 + \beta_2)(\beta_1 - \beta_2)^2 \,\text{sh}\, \gamma t + (\beta_1 + \beta_2)^2 (\beta_1 - \beta_2) \,\text{sh}\, \delta t$$

$$\Delta(t) = (\beta_1 - \beta_2)^2 \,\text{ch}\, \gamma t - (\beta_1 + \beta_2)^2 \,\text{ch}\, \delta t + 4\beta_1\beta_2 \cos \mu t$$

It can be proved that, for $0 < \theta < \pi$, the equation $\Delta_1(t) = 0$ can have only an imaginary root in the strip $-1 < \text{Im}\, z < 0$, and the equation $\Delta(z) = 0$ does not have any roots in this strip. Moreover, for $\theta < \pi/2$ ($\theta_2 < \theta_1 < \pi/2$), the equation $\Delta_1(z) = 0$ does not have any roots in the strip $-1 < \text{Im}\, z < 0$.

For $\alpha < 1$, if $\theta = 2\pi/3$, the function $\Delta_1(z)$ has zeroes at the points $z_0^- = -i/3$, $z_1^- = -2i/3$ and the stress at the point $x = 0$ has the estimate

$$\tau(x) - \tau_0(x) = \tilde{c}_1 x^{-2/3} + \tilde{c}_2 x^{-1/3} + \tilde{\varphi}_3(x)$$

where $\tilde{\varphi}_3(x)$ is a bounded function for $x \geqslant 0$, and $\tilde{c}_1$ and $\tilde{c}_2$ are constants.

When $\pi/2 < 0 < \pi$, by choosing the numbers δ and γ or the numbers β_1 and β_2, we can achieve that the equation $\Delta_1(z) = 0$ has a root in the strip $-1 \leqslant \operatorname{Im} z < 0$. This means that the stress $\tau(x) - \tau_0(x)$ can be bounded as well unbounded at the point $x = 0$.

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